Enhanced PML–like ABCs for layered media transmission line termination

A. Lauer, A. Wien, P. Waldow and I. Wolff
Institute of Mobile and Satellite Communication Techniques,
D–47475 Kamp-Lintfort, Germany, e-mail lauer@imst.de

Abstract—**An Equivalent Circuit (EC) based analytical way is shown to optimize the discrete conductivity profile of PML–like Absorbing Boundary Conditions for layered media transmission line termination. For validation, a two cell microstrip transmission line (Substrate $\varepsilon_r = 12.9$) absorber is presented with $S_{11} <$ -60 dB in a 3D FDTD simulation, which is in excellent agreement with the Equivalent Circuit model's prediction. Therefore a significant reduction of the ABC computation time can be achieved.**

I. Introduction

Perfectly Matched Layer (PML) like boundary conditions [1], [2] have been used for layered media transmission line termination with [3] or without [4] field component splitting.

Herein the usual

$$\sigma(x) = A\left(\frac{x}{\delta}\right)^m \tag{1}$$

PML conductivity profile [1] yields poor absorber performance, if *thin* PML absorbers (up to four elements thickness) are used for fast computation.

Within a one-dimensional equivalent circuit model, parallel plate transmission line absorbers are discussed for homogeneous and layered media (Section II). In Section III an analytical method is developed to obtain a discrete conductivity profile suitable for termination in these cases. Section IV discusses the numerical performance of a realistic 3D microstrip load in comparison to standard conductivity profiles.

II. An Equivalent Circuit Model for Parallel Plate Transmission Line Absorbers

To analyze layered media absorbing boundary conditions, partially filled Parallel Plate Transmission lines sketched in Fig. 1 are a good starting point.

For the air filled transmission line (Type A) the line parameters are

$$L'_0 = \frac{h}{w}\frac{Z_{F,0}}{c_0}, \tag{2}$$

$$C'_0 = \frac{w}{h}\frac{1}{Z_{F,0}c_0}, \tag{3}$$

$$Z_0 = 1/Y_0 = \sqrt{\frac{L'_0}{C'_0}} \tag{4}$$

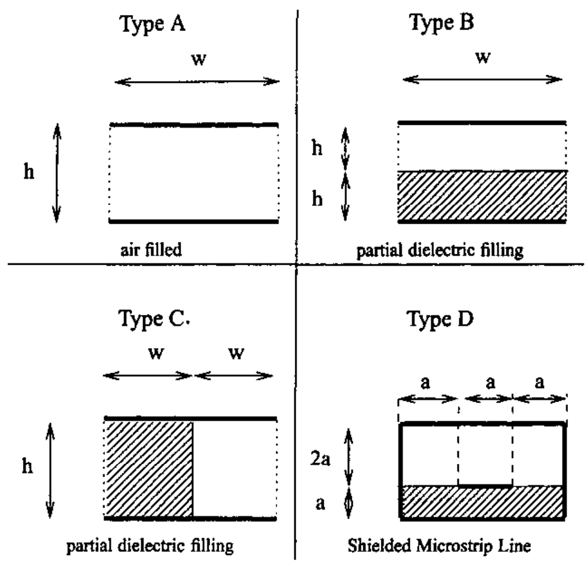

Fig. 1. Transmission line cross sections for discussion of layered media absorbers. Thick lines denote perfectly conducting boundaries (PEC walls), dotted lines magnetic (PMC) walls. Hatches show dielectric filling. The shielded microstrip line (Type D) is used for numerical validation (Section IV) only. In the numerical tests, the substrates' permittivity is $\varepsilon_{r,d} = 12.9$.

with $c_0 \approx 2.997 \cdot 10^8 m/s$, and the characteristic field impedance $Z_{F,0} \approx 120\pi\Omega$.

For the Type B rsp. C transmission lines it is easily seen, that

$$L'_B = 2L'_0, \tag{5}$$

$$C'_B = \frac{\varepsilon_{r,d}}{1+\varepsilon_{r,d}}C'_0, \tag{6}$$

$$Y_B = \sqrt{\frac{C'_B}{L'_B}} = \sqrt{\frac{\varepsilon_{r,d}}{2(1+\varepsilon_{r,d})}}Y_0 = Y_0\sqrt{\varepsilon_{eff,B}}, \tag{7}$$

$$L'_C = \frac{L'_0}{2}, \tag{8}$$

$$C'_C = (1+\varepsilon_{r,d})C'_0, \tag{9}$$

$$Y_C = \sqrt{\frac{C'_C}{L'_C}} = \sqrt{2(1+\varepsilon_{r,d})}Y_0 = Y_0\sqrt{\varepsilon_{eff,C}}, \tag{10}$$

where $\varepsilon_{r,d}$ is the dielectric constant of the partial filling.

The Type C (Type B) arrangement can be looked at as a *series* (*parallel*) circuit of an air-filled and a dielectric filled homoge-

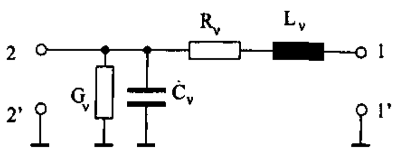

Fig. 2. Equivalent circuit for one FDTD cell within a homogeneously filled Parallel Plate Transmission line including electric and magnetic losses.

neous transmission line. A PML–like FDTD absorbing boundary condition will perform bad, if Y_B and/or Y_C is not accurately achived by connecting an air-filled to a dielectric filled homogeneous medium in a series rsp. parallel sense as mentioned above.

FDTD analysis of a homogeneous Parallel Plate Transmission line is a one dimensional problem yielding an equivalent circuit for one cell shown in Fig. 2. It is

$$L_\nu = \Delta\frac{h}{w}\mu, \quad (11)$$
$$C_\nu = \Delta\frac{w}{h}\varepsilon, \quad (12)$$

where Δ is the spatial resolution in propagation direction.

Outside of the absorber, $G_\nu, R_\nu = 0$. Inside the PML–like[1] absorber it is

$$G_\nu = A_\nu C_\nu = a_\nu \varepsilon_r Y_0, \quad (13)$$
$$R_\nu = B_\nu L_\nu = b_\nu Z_0. \quad (14)$$

If

$$G_\nu = \Delta\frac{w}{h}\sigma_\nu, \quad (15)$$
$$R_\nu = \Delta\frac{h}{w}\sigma_{m,\nu}, \quad (16)$$

with the condition $\sigma/\varepsilon = \sigma_m/\mu$ [1] together with a conductivity profile (1) the equivalent circuit describes classical PML absorbers.

Basic circuit analysis of a chain[2] of absorber cells $\nu = 1..n$ terminated by a *short* circuit (PEC wall) yields for low frequencies $f \to 0$

$$Y_{in,\nu} = Y_0\frac{p_\nu(\varepsilon_r)}{q_\nu(\varepsilon_r)} = Y_0 f_\nu(\varepsilon_r), \quad (17)$$
$$q_1(\varepsilon_r) = 1, \quad (18)$$
$$p_1(\varepsilon_r) = a_1\varepsilon_r + \frac{1}{b_1}, \quad (19)$$
$$q_{\nu+1}(\varepsilon_r) = p_\nu(\varepsilon_r) + q_\nu(\varepsilon_r), \quad (20)$$
$$p_{\nu+1}(\varepsilon_r) = a_\nu\varepsilon_r q_{\nu+1}(\varepsilon_r) + b_\nu q_\nu(\varepsilon_r), \quad (21)$$

giving a recursive algorithm to calculate the input impedance $Y_{in,n}$ of the absorber which ideally has to be

$$Y_{in,n} = Y_0\sqrt{\varepsilon_r}. \quad (22)$$

[1]All nonzero field components vary only in propagation direction, so PML component splitting has no effect here.

[2]Port 2,2' of the cell indexed ν is connected to port 1,1' of the cell $\nu+1$. The short circuit termination is at port 1,1' of cell 1.

It is seen, that

$$p_n(\varepsilon_r) = \sum_{\mu=0}^{n} c_\mu \varepsilon_r^\mu, \quad (23)$$
$$q_n(\varepsilon_r) = \sum_{\mu=0}^{n-1} d_\mu \varepsilon_r^\mu \quad (24)$$

are polynomials depending on ε_r.

In the case of good absorber performance, according to (22)

$$f_n(\varepsilon_r) \approx \sqrt{\varepsilon_r} \quad (25)$$

must be an accurate rational approximation.

For Type B and Type C transmission line absorbers it is straight forward[3] to derive, that

$$Y_{n,in,B} = f_n(\varepsilon_{eff,B})Y_0, \quad (26)$$
$$Y_{n,in,C} = f_n(\varepsilon_{eff,C})Y_0. \quad (27)$$

This means, in the test cases the absorber's performance depends *only* on the accuracy of $f_n(\varepsilon_{eff}) \approx \sqrt{\varepsilon_{eff}}$.

III. Computation of a Discrete Conductivity Profile for Layered Media Absorbers

For arbitrary quasi TEM-mode transmission lines on layered media, it is well known, that for low frequencies

$$\varepsilon_{eff} \in [\varepsilon_r^{min}, \varepsilon_r^{max}], \quad (28)$$

in most cases ε_r^{min} is one.

Thus it is promising to use n cell PML–like absorbers with $f_n(\varepsilon_r) \approx \sqrt{\varepsilon_r}$ accurate for $1 < \varepsilon_r < \varepsilon_r^{max}$ for termination of arbitrary layered media quasi-TEM transmission lines.

For this purpose

$$I_{err} = \int_1^{\varepsilon_r^{max}} \left(p_n(x) - q_n(x)\sqrt{x}\right)^2 g(x)\,dx \quad (29)$$
$$= \int_1^{\varepsilon_r^{max}} \left(\sum_{\mu=0}^{n} c_\mu x^\mu - \sum_{\mu=0}^{n-1} d_\mu x^{\mu+1/2}\right)^2 g(x)\,dx \quad (30)$$

with a weight function $g(x)$ is minimized with regard to c_μ and d_μ. Empirically, $g(x) = x^{-(n+1.25)}$ is found to give a good balance between the error at $\varepsilon_r = 1$ and $\varepsilon_r = \varepsilon_r^{max}$.

So

$$\frac{\partial}{\partial c_\nu} I_{err} = 0, \quad (31)$$
$$\frac{\partial}{\partial d_\nu} I_{err} = 0 \quad (32)$$

[3]By the means of series (Type B) rsp. parallel (Type C) connection of *every single* equivalent circuit element.

$\forall$ ν is a linear equation system to be solved for c_ν, d_ν without problems if d_0 is set to one.

The equivalent circuit $\{a_\nu, b_\nu\}$ and thus the PML absorber conductivity profile is calculated from $p_n(x)$, $q_n(x)$ iteratively by polynomial division as

$$\frac{p_{\nu+1}(x)}{q_{\nu+1}(x)} = a_{\nu+1}x^{\nu+1} + \frac{p_\nu(x)}{q_{\nu+1}(x)}, \tag{33}$$

$$\frac{q_{\nu+1}(x)}{p_\nu(x)} = b_{\nu+1} + \frac{q_\nu(x)}{p_\nu(x)}. \tag{34}$$

IV. Numerical Validation

Figure 3 shows the reflection coefficient of Enhanced PML (EML) Absorbers for Type A, Type B and Type D test structures in dependence of the absorber thickness, Fig. 4 shows standard quadratic conductivity profile PML absorber performance for comparison[4]. The standard PML conductivity profile has been scaled for best vacuum absorption[5].

It is seen, that the EML very much improves layered media absorption.

Figure 5 shows the static reflection coefficient analytically calculated using the one dimensional equivalent circuit model of two cell thick EML rsp. PML absorbers in comparison to FDTD results. The agreement is excellent, which offers the opportunity to look at the static absorber characteristics in a way, that the equivalent circuit "calculates" $\sqrt{\varepsilon_{\text{eff}}}$ from ε_{eff} using a rational approximation. Furthermore it is seen, that EML performs much better than PML if $\varepsilon_{\text{eff}} > 1$.

In Fig. 6, the normalized conductivity profiles for two and four element EML/PML absorbers is shown.

Figure 7 shows the dynamic performance of two cell PML/EML absorbers terminating the test microstrip transmission line. It shows, that the enhanced absorbers perform better in a wide frequency range.

V. Conclusions

A one dimensional Equivalent Circuit model for static analysis and optimization of PML-like Absorbing Boundary Conditions has been presented and verified. It is shown, that simply speaking the absorber's equivalent circuit "calculates" $\sqrt{\varepsilon_{\text{eff}}}$ from ε_{eff} in layered media using a rational approximation. Based upon this, a method has been developed to optimize the discrete PML conductivity profile for best layered media transmission line termination. Numerical tests have shown the EML absorbers' superior performance.

[4]The relatively bad freespace (Type A) performance of quadratic profile PML absorbers at a thickness of two and three cells is beacuse the the PML loss algorithm [1] becomes inaccurate in handling big loss coefficients, e.g. not $\frac{\sigma \Delta_t}{\varepsilon} << 1$.

[5]Scaling to best e.g $\varepsilon_{\text{eff}} = 7$ absorption is not possible, because the absorbers' losses then become too big to be handled by the PML loss algorithm [1].

References

[1] J.-P. Berenger, "A perfectly matched layer for the absorption of electromagnetic waves," *J. Comput. Phys.*, vol. 114, no. 1, pp. 185–200, 1994.

[2] D. S. Katz, E. T. Thiele, and A. Taflove, "Validation and extension to three dimensions of the Berenger PML absorbing boundary condition for FD-TD meshes," *IEEE Microwave Guided Wave Lett.*, vol. 4, pp. 268–270, Aug. 1994.

[3] A. Bahr, A. Lauer, and I. Wolff, "Application of the PML absorbing boundary condition to the FDTD analysis of microwave circuits," in *IEEE MTT-S Int. Microwave Symp.*, vol. 1, (Orlando, FL), pp. 27–30, 1995.

[4] A. Lauer and I. Wolff, "Stable and efficient ABCs for graded mesh FDTD," in *IEEE MTT-S Int. Microwave Symp.*, vol. 2, (Baltimore, MD), pp. 461–464, June 1998.

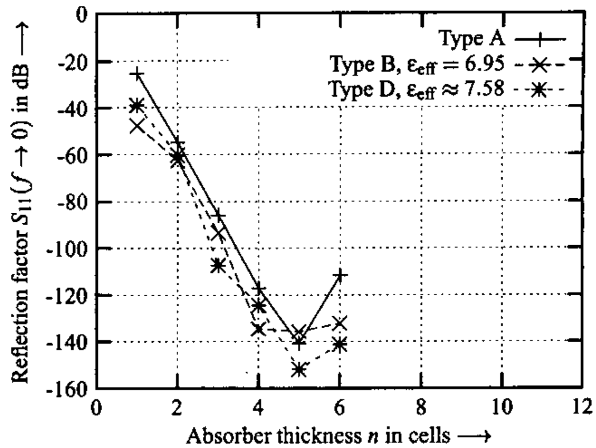

Fig. 3. Reflection coefficient of Enhanced PML Absorbers for different test structures.

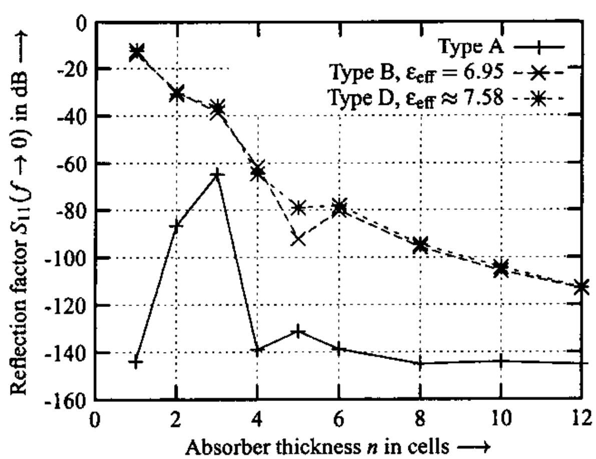

Fig. 4. Reflection coefficient of PML absorbers with quadratic conductivity profile for different test structures. The conductivity profile was scaled for maximum absorption in free space.

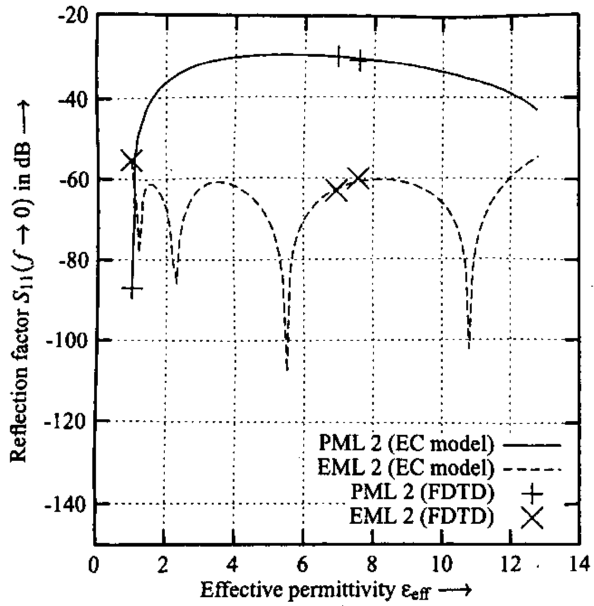

Fig. 5. Static reflection coefficients analytically calculated using the one dimensional equivalent circuit model with the parameter ε_{eff} in comparison to Type A, Type B and Type D simulation results.

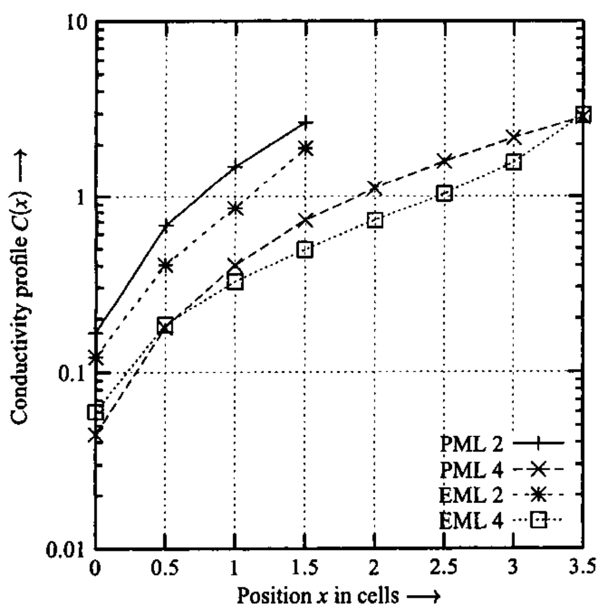

Fig. 6. Conductivity profile $C(x)$ normalized to the free space characteristic impedance.

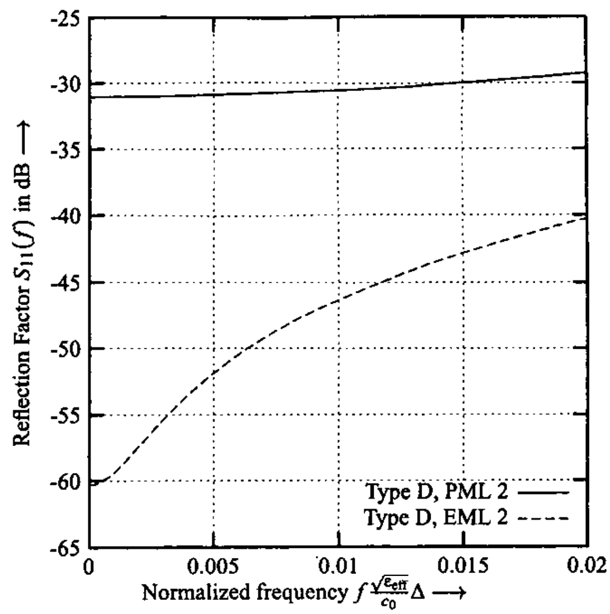

Fig. 7. Dynamic reflection coefficient of two cell PML rsp. EML absorbers for the Type D reference structure.